Wavelet Spectral Finite Elements for Wave Propagation in Composite Plates

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14. ABSTRACT
The principal objectives of the research were to develop and experimentally validate a predictive modeling capability for simulation of wave propagation in 2-D composite plates. During this reporting period, the Wavelet Spectral Finite Element (WSFE) model for composite plates with transverse cracks was developed. Various parametric studies were performed on the developed model to study the wave scattering behavior due to induced transverse cracks. The experimental validation of the developed model is performed at Clarkson University (Dr Ratan Jha?s group). Composite plates were fabricated using aerospace grade carbon fiber prepregs. A scanning laser vibrometer was used to record out-of-plane responses of the plate for Lamb wave actuation. Experimental data and WSFE predictions were shown to have very good correlation for healthy. Development of WSFE models for damaged composite plate (transverse crack and delamination) is complete and the experimental validation at Clarkson University is under progress. The research has resulted in two journal papers and two conference papers so far and three articles are under preparation for journal publication.

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1. Summary of Accomplishments (1 April 2010 – 31 March 2011)

The two principal objectives of this project are to 1) Develop and validate Wavelet Spectral Finite Element (WSFE) models for composite plates with damages such as transverse crack and delamination, and 2) Perform extensive experimental studies for Lamb wave propagation in composite plates with damages. Use time reversal (and other techniques) to obtain characteristic signals for damages. Determine most suited interrogation mechanisms (i.e., wave type, wave mode) for different damages through both simulations and experiments.

During this reporting period, the Wavelet Spectral Finite Element (WSFE) model for composite plates, both health and the damaged plates were developed. The schematic of the composite plate with the transverse cracks is shown in Fig 1

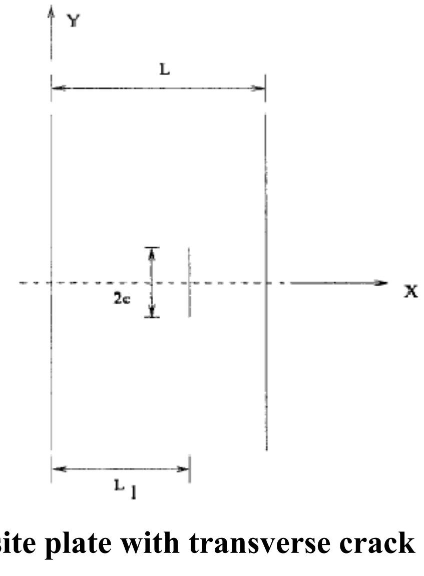


Figure 1: Composite plate with transverse crack of length 2c

The length of the element in the X direction is L and the plate is infinite in the Y direction (although having a finite Y window length L_Y). The crack is located at a distance of L_1 from the left edge of the plate and has a length $2c$ in the Y direction. We consider only the symmetric case for simplification, although the asymmetric case is a straightforward extension. Here, the displacement and slopes on either sides of the crack are different. By enforcing the kinematics on either side of the crack, we can determine the change in the slopes, as well as the moment resultant $M_{xx}(L_1, y)$ at the crack location, which can be related to the flexibility function $\hat{f}_2(y)$

The transverse cracks are modeled using the concept of crack flexibility function. If θ_y denotes the bending flexibility at both sides of the crack, then the slope discontinuity function can be written as

$$\hat{f}_2(y) = \theta(y) M_{xx}(L_1, y)$$

In the above expression, θ_y is the crack flexibility, which is given by

$$\theta(\bar{y}) = (6H/L_y)\alpha_{bb}(\bar{y})F(\bar{y}) \,, \quad \bar{y} = y/L_y \,,$$

where H is the total thickness of the plate, $\alpha_{bb}(\bar{y})$ is function representing dimensionless bending compliance coefficient and $F(\bar{y})$ is a correction function. The function $\alpha_{bb}(\bar{y})$is given by the following relation:

$$\alpha_{bb}(\bar{y}) = \alpha_{bb}^{0} f(\bar{y}) \,,$$

where,

$$\alpha_{bb}^{0} = (1/H)\int_{0}^{h_0} \xi(1.99 - 2.47\xi + 12.97\xi^2 - 23.117\xi^3 + 24.80\xi^4)^2 \, dh \,, \quad \xi = h/H \,,$$

where $h(y)$ is a function representing the shape of the crack and h_0 is the central crack length and location. The plot of crack flexibility as a function of crack location is shown in Fig 2.

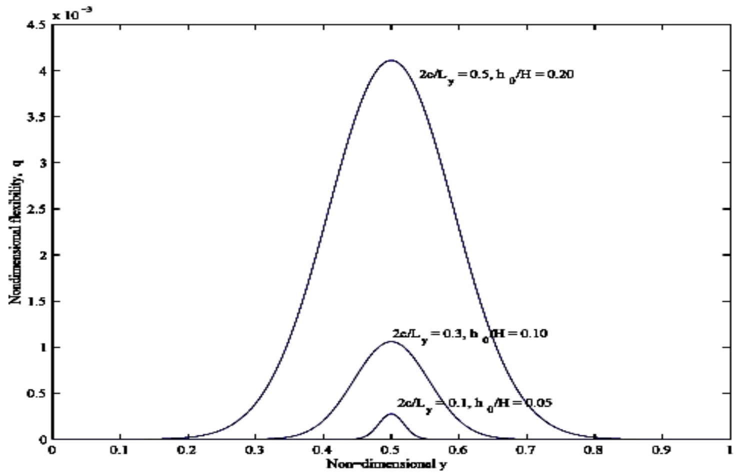
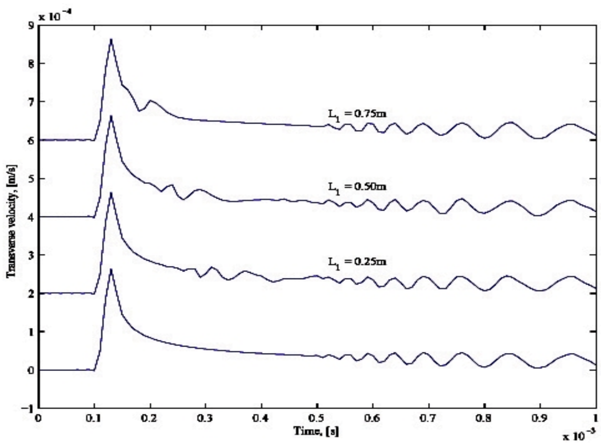


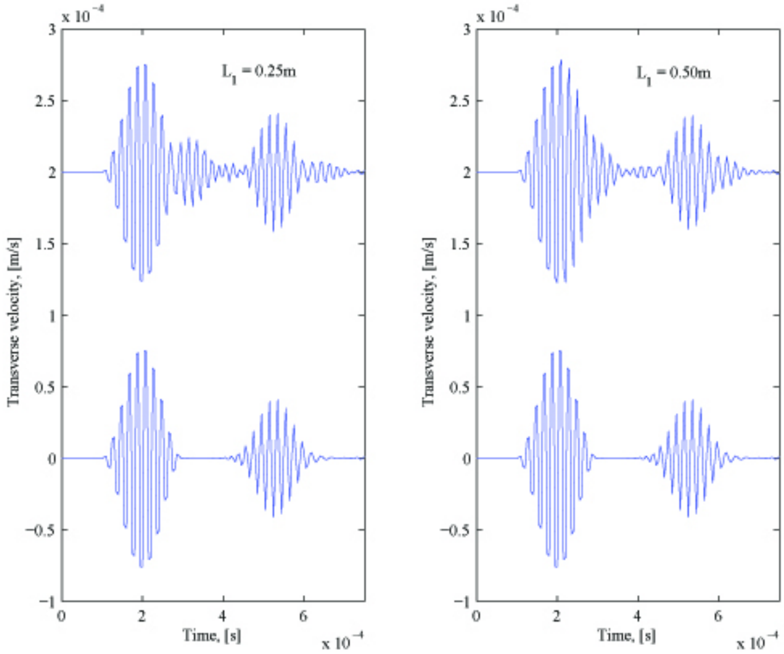
Figure 2: Variation of non dimensional flexibility at different crack location

The plot clearly shows that the crack flexibility seems to have pronounced effect on the crack length and its location. That is, larger the crack length, we see more pronounced is the crack flexibility function.

Next, these crack flexibility function is incorporated into the developed spectral Fem model for plates to study the wave scattering due to the presence of transverse crack. The analysis is performed to both broadband high frequency loading and also the tone burst sinusoidal loading. These plots are shown in Fig. 3.



(a)



(b)

Figure 3: (a) Wave scattering due to broad band pulse (b) Wave scattering due to narrow band pulse

From the plots, we see that the crack size does not alter the peak amplitude and for both these loadings, we see an extra reflections occurring due to the presence of the transverse crack.

EXPERIMANTAL VERIFICATION OF THE HEALTHY SPECTRAL PLATE ELEMENT:

The spectral plate element was developed in year 2009-2010. This developed spectral plate element is experimentally validated at the Clarkson University. Composite plates were fabricated using aerospace grade carbon fiber prepregs. A scanning laser vibrometer was used to record out-of-plane responses of the plate for Lamb wave actuation. The experimental setup is shown in Fig 4.

Figure 4: Experimental setup using scanning Laser Vibrometer

First the Lamb wave speed is correlated using the 3 different modulated tone burst frequency. Table 1 shows the speed predictions from the experiments, which matched well with the Lamb wave curves.

Table 1: Lamb wave speeds of the symmetric mode computed for different tone burst frequencies.

	15kHz, 3.5Cycle	18kHz, 3.5Cycle	20kHz, 3.5Cycle
TOF(ms)	0.0977	0.0928	0.0910
Phase velocity(km/s)	0.614	0.646	0.659

Next, response experiments were performed. Experimental data and WSFE predictions were shown to have very good correlation for healthy plate as shown in Fig. 5. The plots show very close match with the experimental and computed responses.

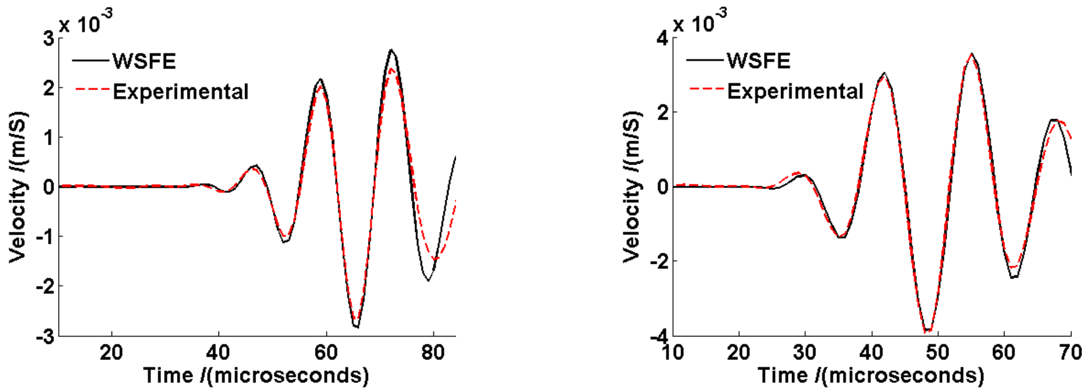


Figure 5: Experimental Validation of the Health Spectral Composite Plate element.

This research on WSFE modeling is performed in close collaboration with Prof. Ratan Jha at Clarkson University (funded through AFOSR). New wavelet spectral plate elements with embedded arbitrary cracks are under development. Here, we will model the region close to crack by conventional FEM, which is reduced using suitable reduced order model. This reduced FEM model will then be coupled to the health spectral plate element, which will be used for wave scattering and damage detection studies. The experimental validation of the transverse crack model, developed here will be experimentally verified at visited Clarkson in the coming year. Prof. Gopalakrishnan visited Clarkson in the year July 2010 and Prof. Jha visited IISc in January 2011 to facilitate extensive interactions between the two research teams.

2. Publications:

Two journal papers and 2-conference paper came out of this research, which are given below.

Ganagadharan, R, Bhat, M R., Murthy, C R L and Gopalakrishnan, S., “Time Reversal Health Monitoring of Composite Plates using Lamb Waves”, *International Journal of Aerospace Innovations (Accepted 2011)*

Ganagadharan, R. Roy Mahapatra, D., Gopalakrishnan, S., Murthy, C R L, and Bhat, M R., “On the sensitivity of elastic waves due to structural damages: time-frequency based indexing method”, *Journal of Sound and Vibrations 320 (4-5), 915-941, 2010*

Gopalakrishnan, S., and Jha, R., " A Wavelet Spectral Element for Composite Plate with Delamination and Transverse Damage", Paper No:AIAA-2010-2901,51stAIAA/ASME/ASCE/AHS/ASCStructures, Structural Dynamics, and Materials Conference, Orlanda, Florida, April 12- 15, 2010

Gopalakrishnan, S., and Jha, R. Inho Kim, and Dulip Widana-Gamage ., “Composite Delamination Detection Using Wavelet Spectral Finite Element and Damage Force Indicator Method”, 2901,51stAIAA/ASME/ASCE/AHS/ASCStructures, Structural Dynamics, and Materials Conference, Denver, Colorado, April4-7, 2011